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On the frequency response of moderately thick simply supported rectangular plates with arbitrary lamination

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Abstract

A here-to-fore unavailable analytical solution to the boundary-value problem of free vibration response of arbitrarily laminated moderately thick rectangular plates, subjected to an admissible boundary condition is presented. A novel boundary continuous solution approach is employed to solve the highly coupled governing partial differential equations that arise from the implementation of the Yang–Norris–Stavsky (1966) theory incorporating first-order shear deformation, and rotary and in-plane inertias. The numerical results presented for various parametric effects to study first lowest seven eigenvalues, and corresponding eigenvectors, can be capitalized as bench-marks for a future comparison. An eight-node isoparametrics shear-flexible element is considered to validate its performance with respect to the present solutions. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

A tremendous growth of advanced fiber reinforced laminated composites as primary structural materials in aerospace, aircraft, automotive, marine, submarine, and other industrial applications, has impelled a growing interest among the researchers in the quest to understand the structural response of these laminates, in general, and particular in the field of dynamics. Even for a simplest geometry and lamination, the analysis of these laminates is loaded with such complexities as introduced by couplings among the stiffness terms in the governing partial differential equations as well as in boundary conditions. This coupling phenomenon plays an important role in selecting solution methodology. In approximate domain of analysis, use of the Galerkin, Rayleigh–Ritz, Collocation, Finite Element, Finite Element Difference, etc., may not pose difficulties due to the coupling terms as they do in the domain of an analytical approach. As a result analytical solution approaches are more conspicuous in the current literature than by their absence. However, an approximate solution users usually validate their methodology comparing some bench-mark prob-

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lems obtained using analytical approach. Therefore, the importance of an analytical based solution remains in the field of an engineering requirement along with the difficulties in obtaining them. The difficulty is experienced in all three plate theory: (i) Classical Lamination Theory (CLT)—based on Kirchhoff's hypothesis, (ii) First Order Shear Deformation Theory (FSDT)—based on Reissner (1944) and Mindlin (1951) theory, and (iii) Higher Order Shear Deformation Theory (HOSDT)—based on linear variation of shear deformation across the thickness. Bert and Maybberry (1969) appear to be the first to be credited for the analysis of fully clamped rectangular laminated plates with CLT formulation using the approach of the Ritz method. Craig and Dawe (1986) have used the Ritz and finite strip methods to analyse vibration problems of symmetric angle-ply rectangular plates. Following the approximate solution technique, Ashton (1970) has reported the analysis of all edge clamped anisotropic plate, in which the deflection function is assumed in the form of series involving beam characteristic shapes. Use of approximate solution techniques are also found in the works of Chamis (1969), Leissa and Narita (1989), Liew and Lam (1991), Chow et al. (1992), Hung et al. (1993a, b), Baharlou and Leissa (1993), Chai (1994), Liew and Lim (1995), and Liew et al. (1996). Using boundary discontinuous double Fourier series solution functions, Whitney (1969, 1979), and Whitney and Leissa (1970) have presented analytical solution to rectangular plates with anti-symmetric cross-ply and angle-ply lamination for various boundary conditions. Recently, Librescu et al. (1989) have given solution to free vibration problems of thin plates for various boundary conditions. In their analysis they have considered anti-symmetric cross-ply and angle-play lamination, only.

In the domain of FSDT, majority of the works are found with cross-ply and angle-ply lamination (symmetric or anti-symmetric). Srinivas et al. (1970) are the first to obtain an exact Navier-type solution to the problem of free vibration of homogeneous and cross-ply thick plates. Bert and Chen (1978) have also demonstrated the Navier-type solution but for free vibration problems of anti-symmetric angle-ply rectangular plates. Palardy and Palazotto (1990) using the Levy-type approach, have presented a solution to rectangular plates with symmetric cross-ply lamination form various boundary conditions, with two opposite-edges being invariably simply supported (Navier-type). Librescu et al. (1989) have also used the Levy-type approach and given solution to anti-symmetric cross-ply and angle-ply lamination to vibration response of rectangular plate problems. Chaudhuri and Kabir (1994) has given analytical solution to free vibration response of cross-ply laminated plates with simply supported and all-edge clamped boundary conditions. Using a boundary continuous generalized Navier's approach, Kabir and Chaudhuri (1994) have presented an analytical solution to free vibration problems of arbitrarily laminated rectangular plates with fully clamped boundary conditions at all edges. It is worth mentioning here that the solution functions selected for this particular boundary condition, may not be suitable for other cases. Interested readers may find an extensive literature survey in the recent works of Liew et al. (1995) on mainly FSDT-based formulations. To the best of the knowledge of the author, no free vibration response is studied analytically for a simply supported (appears to be a simple one) plates with arbitrary lamination having FSDT in consideration, an important mile-stone required to be established towards the realm of analytical solution.

In regard to the HOSDT-based analytical field, most of the solutions are performed for cross-ply and angle-ply lamination, as can be seen in the works of Reddy (1984) and Librescu and Khdeir (1988). In this field also, to the best of the knowledge of the author, no analytical solutions to free-vibration response for arbitrary lamination is reported. However, in this paper the scope

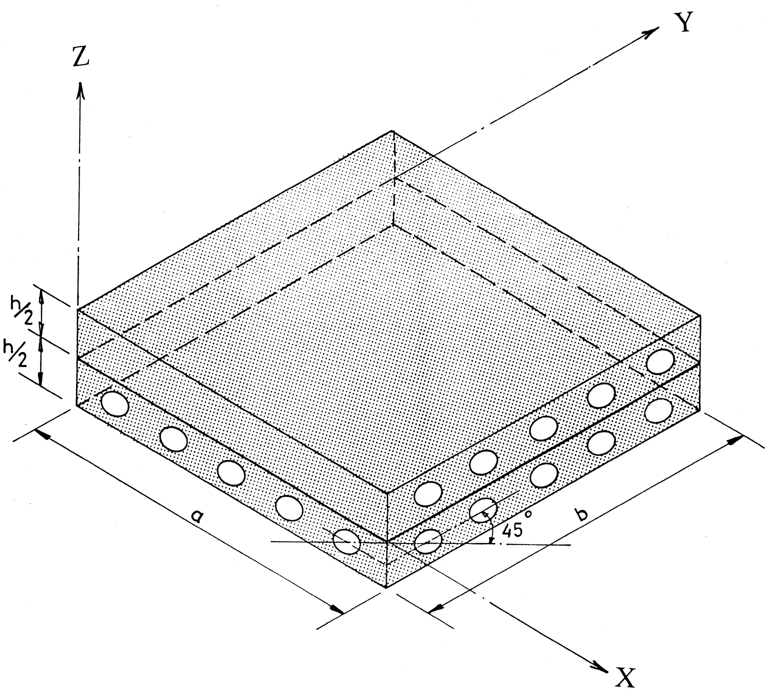

Fig. 1. A typical arbitrary laminated rectangular plate.

of the development will be limited to only FSDT-based formulation, as a first step towards the goal. Therefore, the first objective is to develop an analytical solution to the free-vibration response of a rectangular plate with arbitrary laminations using FSDT. The second objective is to compare the present results with the results obtained from the finite element method.

1.1. Theoretical formulation

Figure 1 illustrates a geometry of a laminated rectangular plate with arbitrary laminations. h indicates total thickness of the plate while $h^{(k)}$ denotes thickness of kth layer. The reference surface is placed at middle height of the thickness h. The reference coordinate axes (x, y, z) are placed at the reference surface. The dimensions a and b are along x- and y-axes, respectively. The fiber orientation is measured from x-axis, using right-hand thumb rule. The deformation characteristic behavior of a arbitrary laminated plate using FSDT is defined as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ \text{sym} & [D] \end{bmatrix}$$

and

$$\{Q\} = [\bar{A}]\{k^1\} \tag{1}$$

where

$$N^{\mathrm{T}} = \{N_x, N_y, N_{xy}\} \tag{2}$$

$$M^{\mathrm{T}} = \{M_x, M_y, M_{xy}\} \tag{3}$$

$$\{\varepsilon^0\}^{\mathrm{T}} = \{\varepsilon^0_x, \varepsilon^0_y, \varepsilon^0_{xy}\} \tag{4}$$

$$\{k^0\}^{\mathrm{T}} = \{k^0_x, k^0_y, k^0_{xy}\} \tag{5}$$

$$\{Q\}^{\mathrm{T}} = \{Q_x, Q_y\} \tag{6}$$

$$\{k^1\}^{\mathrm{T}} = \{k^1_x, k^1_y\} \tag{7}$$

$\{N\}$ represent in-place force resultants, while $\{M\}$ represent force couple resultants. $\{Q\}$ represent resultants of transverse shear stress. Defining u_i ($i = 1$–5) as displacement of middle surface along x-, y-, and z-axes and rotation of normals about y and x, respectively, $\{\varepsilon^0\}$, $\{k^0\}$ and $\{k^1\}$ can be expressed as

$$\varepsilon^0_x = \frac{\partial u_1}{\partial x} \tag{8}$$

$$\varepsilon^0_y = \frac{\partial u_2}{\partial y} \tag{9}$$

$$\varepsilon^0_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \tag{10}$$

$$\sigma^0_x = \frac{\partial u_4}{\partial x} \tag{11}$$

$$k^0_y = \frac{\partial u_5}{\partial y} \tag{12}$$

$$k^0_{xy} = \frac{\partial u_4}{\partial y} + \frac{\partial u_5}{\partial x} \tag{13}$$

$$k^1_x = \frac{\partial u_3}{\partial y} + u_5 \tag{14}$$

$$k^1_y = \frac{\partial u_3}{\partial x} + u_4 \tag{15}$$

$[A]$, $[B]$, $[D]$ and $\bar{A}$ are defined in Reddy and Miravete (1995).

Using the principle of virtual work, the governing equilibrium equations in terms of stress resultant are defined as

$$\frac{\partial N_1}{\partial x} + \frac{\partial N_{12}}{\partial y} = I_1 \tag{16}$$

$$\frac{\partial N_{12}}{\partial x} + \frac{\partial N_2}{\partial y} = I_2 \tag{17}$$

$$\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} = I_3 \tag{18}$$

$$\frac{\partial M_1}{\partial x} + \frac{\partial M_2}{\partial y} = I_4 \tag{19}$$

$$\frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} = I_5 \tag{20}$$

where I_i ($i = 1$–5) are mass inertia terms defined as

$$I_1 = \rho_1 u_{1,tt} + \rho_2 u_{4,tt} \tag{21}$$

$$I_2 = \rho_1 u_{2,tt} + \rho_2 u_{5,tt} \tag{22}$$

$$I_3 = \rho_1 u_{3,tt} \tag{23}$$

$$I_4 = \rho_2 u_{1,tt} + \rho_3 u_{4,tt} \tag{24}$$

$$I_5 = \rho_2 u_{2,tt} + \rho_3 u_{5,tt} \tag{25}$$

in which

$$(\rho_1, \rho_2, \rho_3) = \sum_{k=1}^{N} \int_{h^{(k-1)}}^{h^k} \rho^{(k)}(1, z, z^2)\,\mathrm{d}z \tag{26}$$

$\rho^{(k)}$ unit mass of kth layer. The governing differential equation may be written in terms of displacement functions and their derivatives:

$$\mathbf{Lu} = \mathbf{f} \tag{27}$$

where

$$L_{ij} = L_{ji}; \quad i,j = 1,\ldots,5 \tag{28}$$

$$\mathbf{u}^{\mathrm{T}} = \{u_1, u_2, u_3, u_4, u_5\} \tag{29}$$

$$\mathbf{f}^{\mathrm{T}} = \{I_1, I_2, I_3, I_4, I_5\} \tag{30}$$

For the sake of brevity L_{1j} elements are given

$$L_{11} = A_{11}\partial_x^2 + 2A_{16}\partial_x\partial_y + A_{66}\partial_y^2 \tag{31}$$

$$L_{12} = A_{16}\partial_x^2 + (A_{12} + A_{16})\partial_x\partial_y + A_{26}\partial_y^2 \tag{32}$$

$$L_{13} = 0 \tag{33}$$

The following non-Navier- and non-Levy-type boundary conditions are considered:

$$\text{at } x = 0, a \qquad u_1 = u_3 = u_4 = 0 \qquad M_x = 0 \tag{34}$$

$$\text{at } y = 0, b \qquad u_2 = u_3 = u_4 = 0 \qquad M_y = 0 \tag{35}$$

$$\text{at all edges} \qquad N_{xy} = 0 \tag{36}$$

The above boundary conditions are referred to as SS2-type boundary conditions (Hoff and Rehfield, 1965; Kabir, 1996). The goal here is to solve eqn (27) in conjunction with the edge conditions as stipulated in eqns (34)–(36).

1.2. Analytical solution to governing equations

The assumed solution functions for the stated problem are selected in terms of double Fourier series in the following form:

$$u_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(1)} \sin(\alpha_m x)\cos(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{37}$$

$$u_2 = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(2)} \cos(\alpha_m x)\sin(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{38}$$

$$u_3 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(3)} \sin(\alpha_m x)\sin(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{39}$$

$$u_4 = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(3)} \cos(\alpha_m x)\sin(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{40}$$

$$u_5 = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} A_{mn}^{(5)} \sin(\alpha_m x)\cos(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant b \qquad 0 \leqslant y \leqslant b \tag{41}$$

where $A_{mn}^{(i)}$ are Fourier constants. α_m and β_n are defined as $m\pi/a$ and $n\pi/b$, respectively. The assumed displacement functions completely satisfy the geometric boundary conditions, a priori, before their substitution into the governing partial differential equations. It may be noticed that natural boundary conditions pose some difficulties in satisfying some ordinary discontinuities existing in some derivatives. In order to clarify more on the above phenomenon, function u_1 is considered. The first derivative of u_1 with respect to y can easily be obtained, however, the second derivative may not be:

$$u_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(3)} \sin(\alpha_m x)\cos(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{42}$$

$$u_{1,y} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \beta_n A_{mn}^{(3)} \sin(\alpha_m x)\sin(\beta_n y)\,\mathrm{e}^{i\bar{\lambda}t} \qquad 0 \leqslant x \leqslant a \qquad 0 < y < b \tag{43}$$

Therefore, $u_{1,y}$ is having some discontinuities at $y = (0, b)$. So its further differentiation is not possible before its substitution into the partial differential equations. At this stage $u_{1,yy}$ is obtained in the following manner as suggested by Hobson (1926), Whitney and Leissa (1970), Whitney (1970), Chaudhuri (1989), Kabir (1996), etc.

$$u_{1,yy} = \frac{1}{2}\sum_{m=1}^{\infty} a_1^m \sin(\alpha_m x)\, e^{i\bar{\lambda}t} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} [-\beta^2 A_{mn}^{(1)} + \omega_n^{(1)} a_1^m + \omega_n^{(2)} a_2^m]\,\mathrm{e}^{i\bar{\lambda}t} 0 \leqslant x \leqslant a 0 \leqslant y \leqslant b \tag{44}$$

where a_1^m are constants arising due to the discontinuities of the derivatives of the functions. These constants are related to the functions at edges in the following form:

$$\begin{Bmatrix} a_1^m \\ a_2^m \end{Bmatrix} = \frac{4}{ab}\int_0^a \begin{bmatrix} \partial_y & -\partial_y \\ -\partial_y & \partial_y \end{bmatrix} \begin{Bmatrix} u_1(x,b) \\ u_1(x,b) \end{Bmatrix} \sin(\alpha_m x)\,\mathrm{d}x \tag{45}$$

The above procedure is applied to the situation, whenever discontinuities in derivatives arise. This then paves the way for substitution of derivatives of the functions into the governing equations. However, another difficulty arises at governing equations level, after substituting all the necessary derivatives of the functions. This happens due to the occurrence of, as for example in the first equation of eqn (28), terms $\sin(\alpha_m x)\cdot\cos(\beta_n y)$, $\cos(\alpha_m x)\cdot\sin(\beta_n y)$, etc. This apparently provides a notion that one will get more equations than unknowns. This situation is managed by expanding [in case of the first equation of eqn (28)] $\cos(\alpha_m x)\cdot\sin(\beta_n y)$ in the following form as suggested by Green (1944), Green and Hearmon (1945) and Kabir and Chaudhuri (1994).

$$\cos(\alpha_m x), \sin(\beta_n x) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} v_{rm}\eta_{sn}\sin(\alpha'_r x)\cos(\beta\delta y) \qquad 0 \leqslant x \leqslant a \qquad 0 \leqslant y \leqslant b \tag{46}$$

where

$$\alpha'_r = \frac{\pi r}{a} \quad \text{and} \quad \beta'_s = \frac{\pi s}{b} \tag{47}$$

$$v_{rm} = \begin{Bmatrix} \dfrac{4r}{\pi(p^2-m^2)} \\ 0 \end{Bmatrix} \begin{matrix} m+r = \text{odd} \\ m+r = \text{even} \end{matrix} \qquad m \neq r \tag{48}$$

$$\eta_{sn} = \begin{Bmatrix} \dfrac{4_s}{\pi(s^2-n^2)} \\ 0 \end{Bmatrix} \begin{matrix} n+s = \text{odd} \\ n+s = \text{even} \end{matrix} \qquad n \neq s \tag{49}$$

The above manipulation in governing equations and natural boundary conditions whenever necessary, yields finally as many equations as the unknowns. Finally, the set of equations are casted in the form suitable for eigenvalue problems. A FORTRAN Program AFSANA-VIB (A Fourier Series ANAlysis—VIBration) is developed to code the theory. The code is linked with ISML-subroutine for solving eigenvalue problems.

2. Numerical results and discussion

In what follows, the numerical results pertaining to the first seven lowest natural frequencies and corresponding eigenvectors for a 0°/45° lamination sequence of a rectangular plate, are

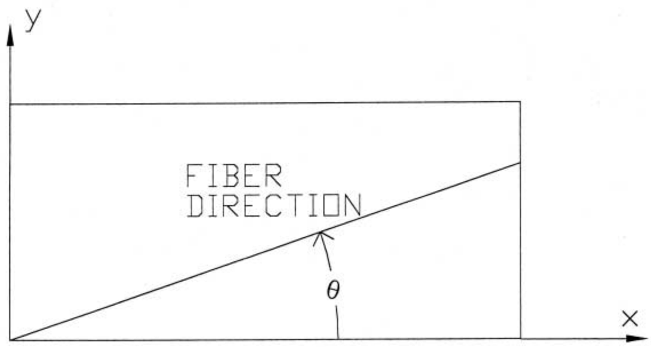

Fig. 2. Fiber orientation scheme.

presented. The angle of a lamina is about z-axis measured from x-axis, and is shown in Fig. 2. The following material properties for a graphite/epoxy orthotropic lamina is considered:

$$E_1 = 76\,\text{GPa}$$
$$E_2 = 5.5\,\text{GPa}$$
$$E_{12} = E_{13} = 1.2\,\text{GPa}$$
$$E_{23} = 1.5\,\text{GPa}$$
$$v_{12} = v_{13} = 0.28$$
$$v_{23} = 0.34$$

where E_1 and E_2 are major and minor moduli of elasticity, respectively. G_{12} represents in-plane shear modulus, while G_{13} and G_{23} denote transverse shear moduli. v_{ij} are Poisson's ratios. A shear correction factor, $K_1' = K_2' = 5/6$, is used (Bert and Chen, 1978). The normalized natural frequencies are defined as

$$\lambda_i = \bar{\lambda}_i a^2(\rho/E_2)^{1/2}/h$$
$$i = 1\text{–}7$$

where

$$\bar{\lambda}_1 < \bar{\lambda}_2 < \cdots < \bar{\lambda}_7$$

The accuracy of any series solution is usually ascertained by studying the convergence of unknown parameters. To such a quest of study, Figs 3–6 plot numerically lowest seven natural frequencies for various span-to-depth, side-to-width ratios. Figure 3 presents plots for $b/a = 1$, and $a/h = 10$. First five lowest frequencies converge for as low as $m = n = 3$, showing excellent convergence with almost no variations. However, λ_6 shows smooth convergence for $m = n > 5$, almost similar nature as found in λ_7 as well.

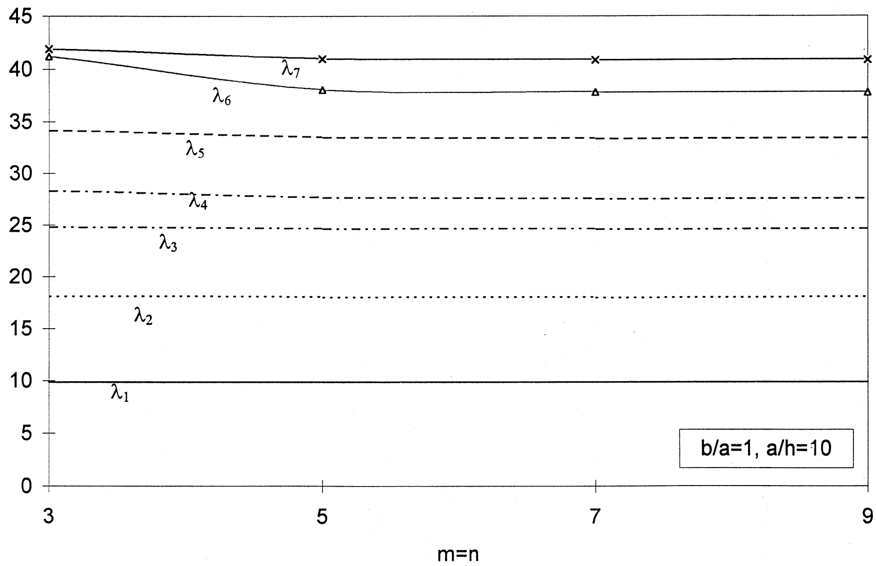

Fig. 3. Convergence of normalized eigenvalues, λ_i (i = 1–7), for a plate with $b/a = 1$ and $a/h = 0$.

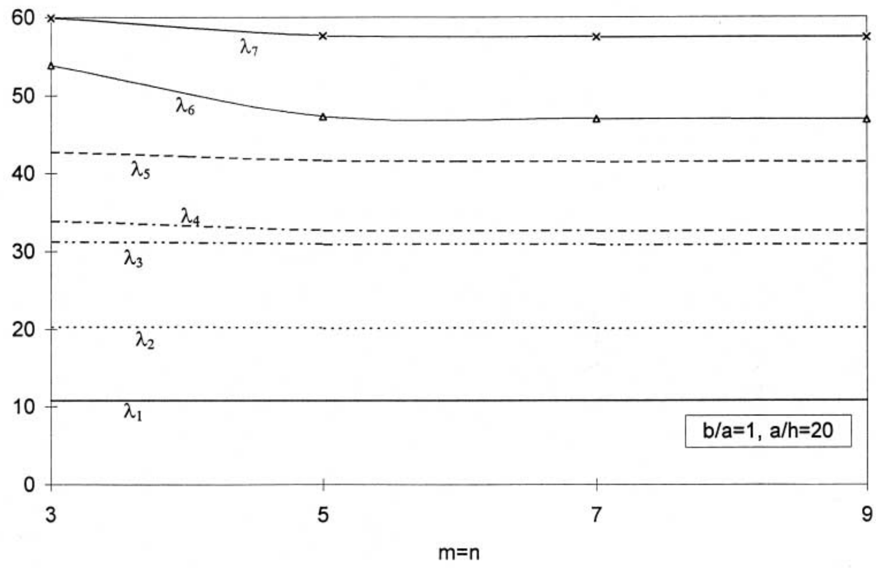

Fig. 4. Convergence of normalized eigenvalues, λ_i (i = 1–7), for a plate with $b/a = 1$ and $a/h = 20$.

Similar plots are presented in Fig. 4 for $b/a = 1$, and $a/h = 20$, showing the convergence nature the same as of $a/h = 10$. However, the same is not achieved for the case of $b/a = 2$, and $a/h = 10$, with exceptions of $\bar{\lambda}_1$, $\bar{\lambda}_2$, $\bar{\lambda}_3$ and $\bar{\lambda}_5$. $\bar{\lambda}_4$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ converge with $m = n > 7$. The same nature as of Fig. 5 is found for the case of $b/2 = 2$, and $a/h = 20$ (Fig. 6). Variations of $\bar{\lambda}_i$ (i = 1–7) with respect

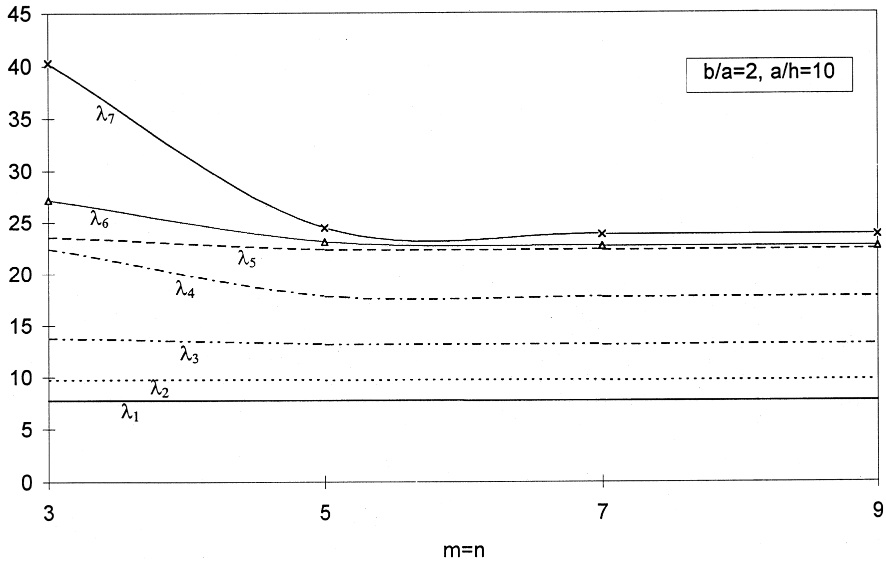

Fig. 5. Convergence of normalized eigenvalues, λ_i (i = 1–7), for a plate with $b/a = 2$ and $a/h = 10$.

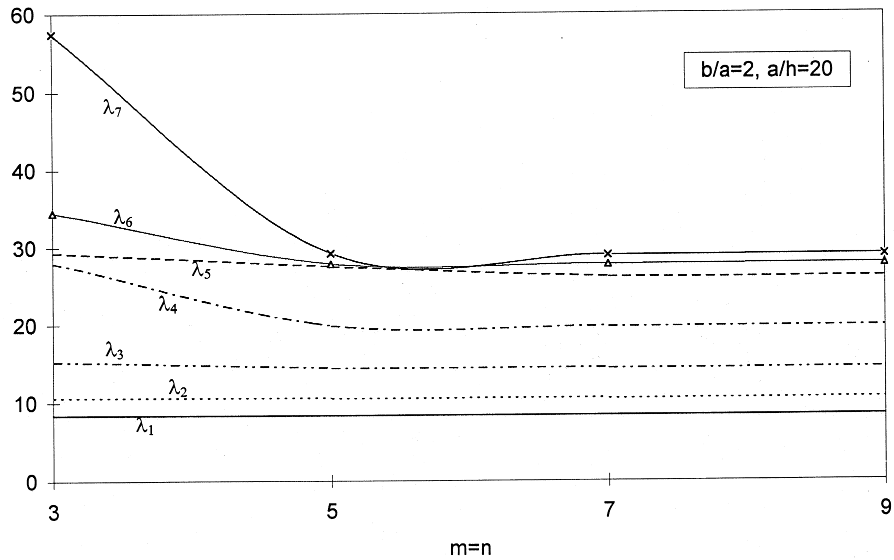

Fig. 6. Convergence of normalized eigenvalues, λ_i (i = 1–7), for a plate with $b/a = 2$ and $a/h = 20$.

to a/h are shown for $(a/b) = 1$ and $(b/a) = 2$, respectively, in Figs 7 and 8. The thickness effects are more prominent for higher natural frequencies for both $(b/a) = 1$ and 2. Eigenvectors are plotted in Figs 9–15 in which $u_3(i,j)$ indicate (i,j) component of eigenvectors related to u_3 dis-

Table 1
A comparison of normalized λ_i (i = 1–7) with respect to the present and finite element methods

λ	$a/h = 10$		$a/h = 20$	
	Analytical	FEM	Analytical	FEM
λ_1	9.85	9.85	10.78	10.56
λ_2	17.92	17.69	20.08	19.58
λ_3	24.54	23.29	30.76	27.19
λ_4	27.49	27.86	32.52	32.61
λ_5	33.34	32.30	41.40	38.96
λ_6	37.80	38.37	46.92	47.89
λ_7	40.39	40.93	57.46	53.02

$b/a = 1.0$; $m = n = 7$ (for analytical method)

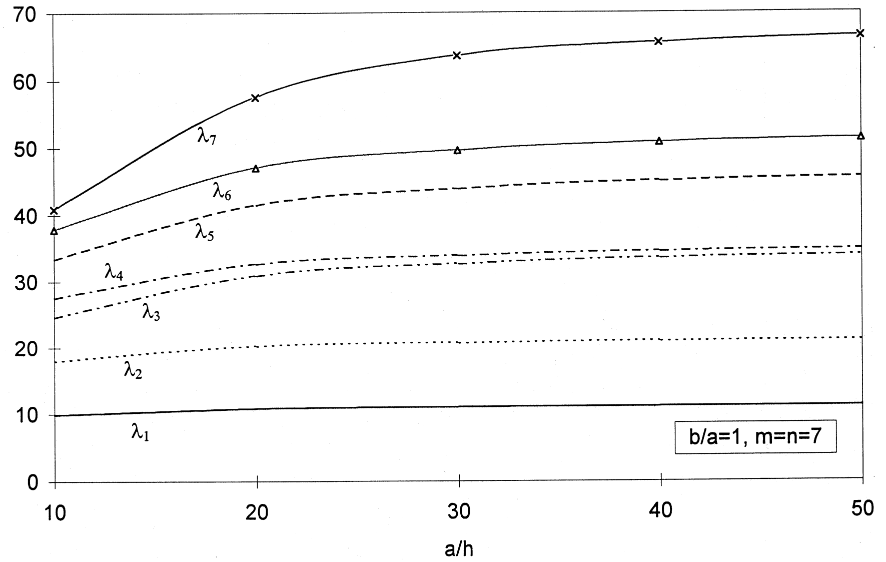

Fig. 7. Variations of normalized eigenvalues, λ_i (i = 1–7), for a plate with $b/a = 1$ and $m = n = 7$.

placement. Table 1 presents a comparison of the present results with finite element results. An isoparametric eight-node finite element based on FSDT formulation is considered. The element uses the reduced integration scheme. The formulation and the development of the element are not presented here for the sake of brevity, and interested readers may find in Bathe (1982), Zienkiewicz et al. (1971), and Hinton and Owen (1977) more details. The comparison is presented for $a/h = 10$

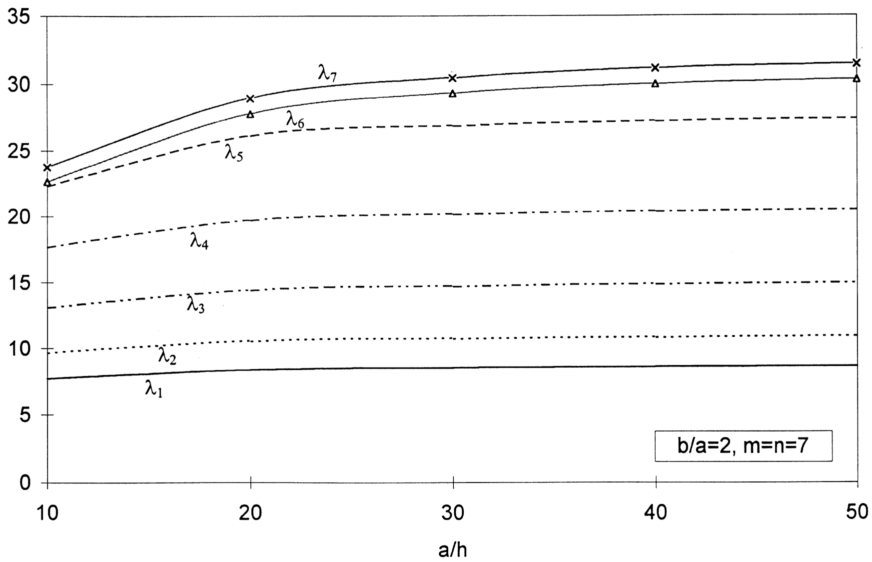

Fig. 8. Variations of normalized eigenvalues, λ_i ($i = 1$–7), for a plate with $b/a = 2$ and $m = n = 7$.

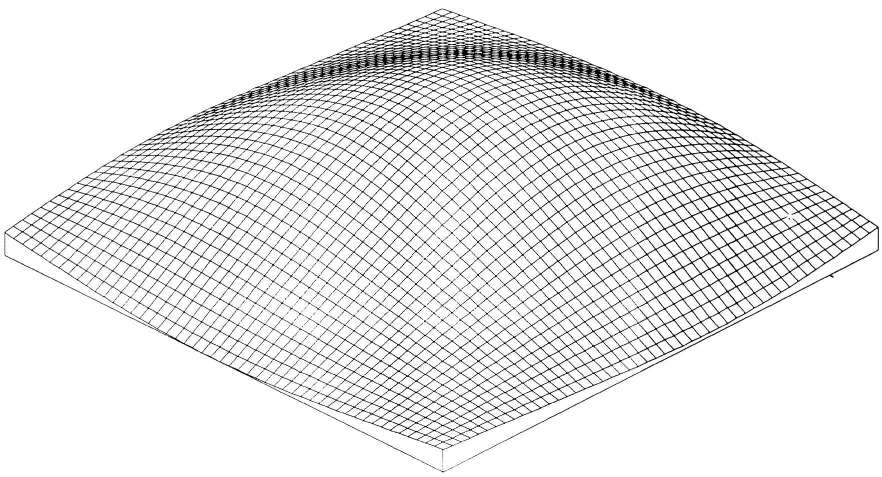

Fig. 9. A mode shape $u_{3(1,1)}$ for a plate with $b/a = 1$ and $a/h = 10$.

and 20, with $b/a = 1$, and $m = n = 7$. The finite element results are more agreeable to the analytical one for the case of $a/h = 10$ than $a/h = 20$. The reason could be due to the use of reduced integration scheme in the finite element development which has possibly made the element more flexible than required.

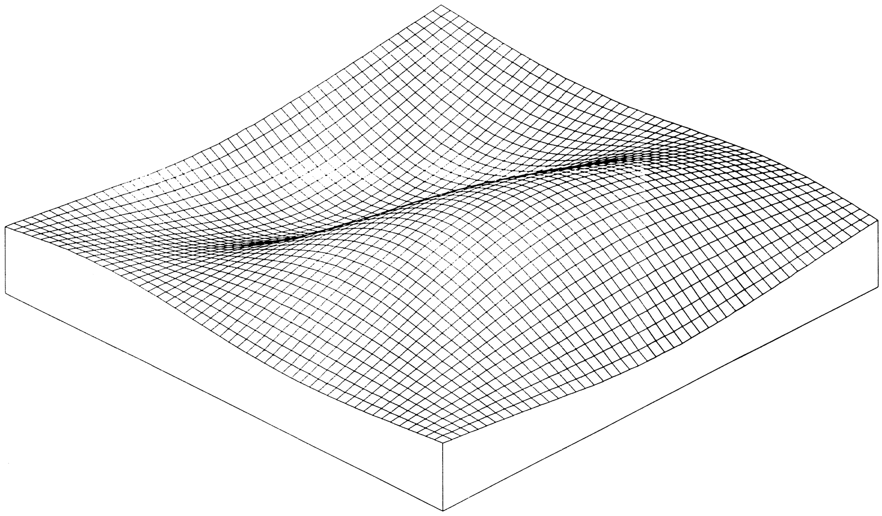

Fig. 10. A mode shape $u_{3(1,2)}$ for a plate with $b/a = 1$ and $a/h = 10$.

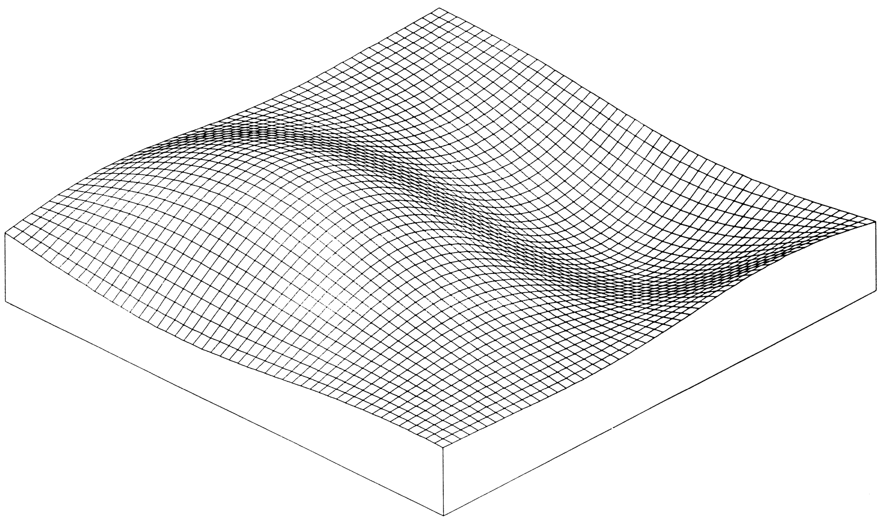

Fig. 11. A mode shape $u_{3(2,1)}$ for a plate with $b/a = 1$ and $a/h = 10$.

3. Conclusion

An analytical solution to the free vibration response of shear flexible rectangular plates with arbitrary lamination is presented, based on boundary continuous double Fourier series solution

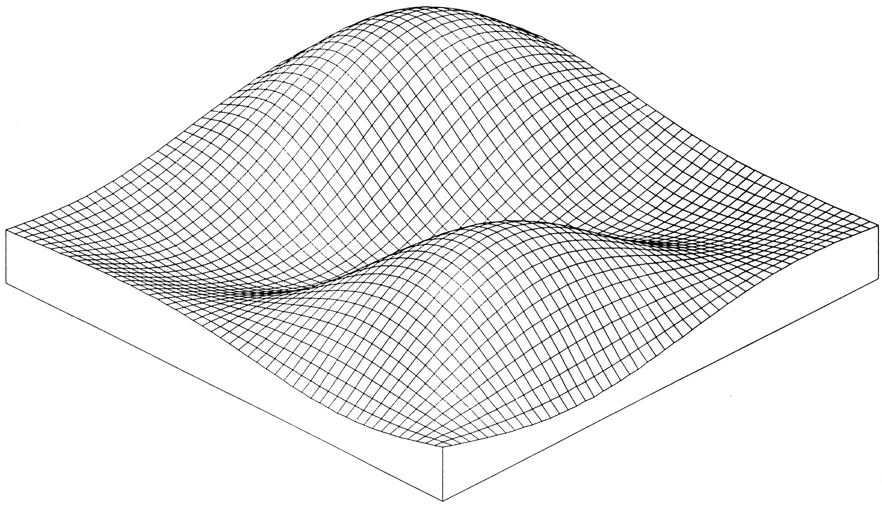

Fig. 12. A mode shape $u_{3(2,2)}$ for a plate with $b/a = 1$ and $a/h = 10$.

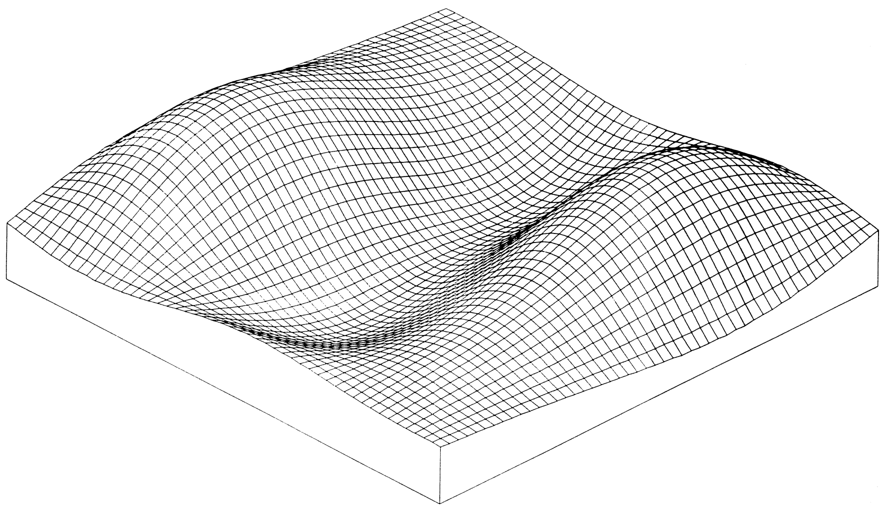

Fig. 13. A mode shape $u_{3(1,3)}$ for a plate with $b/a = 1$ and $a/h = 10$.

functions. A computer program AFSANA-VIB is developed to obtain numerical results. The numerical results presented in the form of eigenvalue and eigenvectors can be used as threshold references for future comparisons.

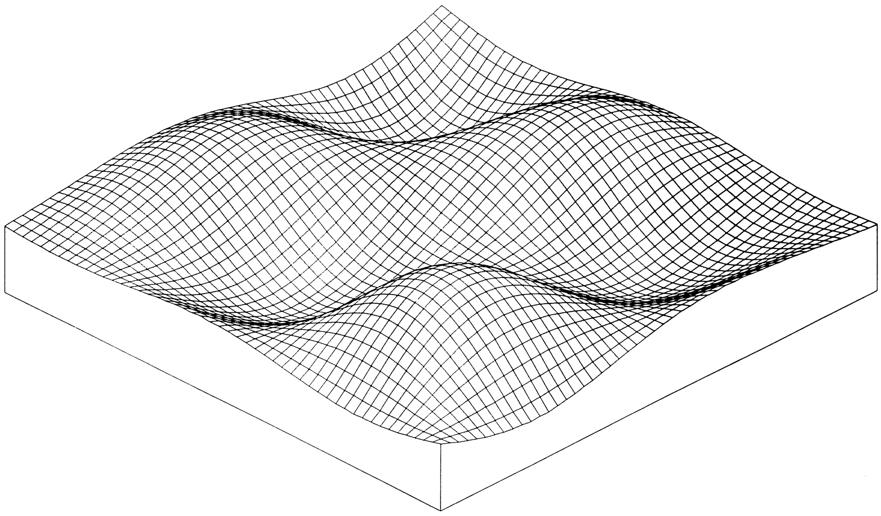

Fig. 14. A mode shape $u_{3(2,3)}$ for a plate with $b/a = 1$ and $a/h = 10$.

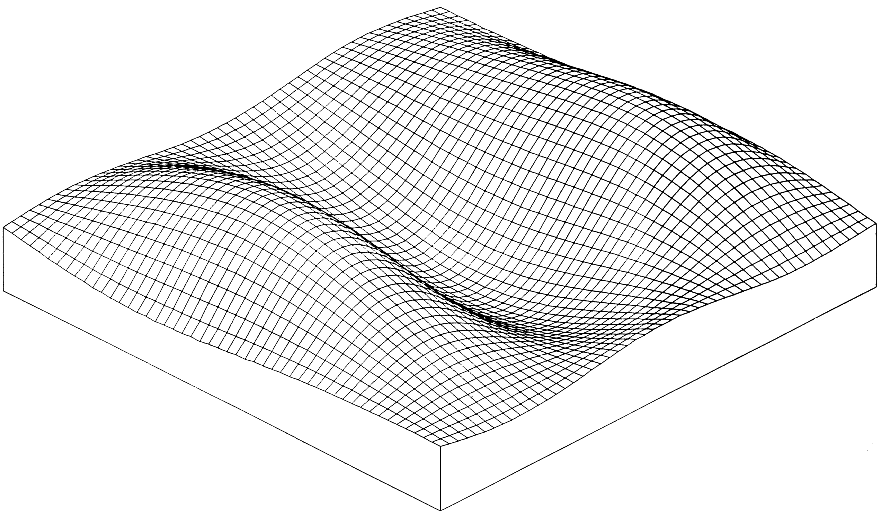

Fig. 15. A mode shape $u_{3(2,3)}$ for a plate with $b/a = 1$ and $a/h = 10$.

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